Scheme of Studies

MS Mathematics
&
Ph.D. Mathematics

Department of Mathematics

Kohsar University Murree

December 2022

MS MATHEMATICS PROGRAMME

The MS degree in Mathematics focuses on strengthening the ability of a student in mathematical reasoning and logic thinking. Students in this program prepare themselves either for their further improvement in the field of Mathematics or for jobs in academic, industrial, business and government/non-government organizations. The program offers a flexible framework including scheme of courses covering major areas of Mathematics like pure mathematics, Computational Mathematics, applied mathematics and financial mathematics.

Admission Criteria

- Sixteen years of schooling or 4-year education after HSSC/F.A./F.Sc./Grade 12, equivalent will be required for admission in the master's degree.
- Required aggregated CGPA of not less than 2.50 out of 4.00 under semester system or 45% of the marks secured under annual examination system or its equivalent in the field of study.
- GAT-General conducted by the National Testing Service with a minimum cumulative score of 50%.

  or

  Entry test (admission test) will be conducted by the university itself through committee approved for the purpose or by engaging renowned testing serviceprovider and it is mandatory to qualify with 60% marks.
- Merit will be determined on the basis of marks obtained in the last degree/certificate.

Ph.D. MATHEMATICS PROGRAMME

Mathematicians use theoretical and computational methods to solve a wide range of problems from the most abstract to the very applied. KUM's mathematics graduate students work in many branches of pure and applied mathematics. The PhD program trains students to operate as research mathematicians. The focus of the program is on substantial mathematical research leading to the PhD dissertation. Students also develop their skills in presenting and teaching mathematics and its applications. The Department of Mathematics is committed to preparing students to meet the demand and developing careers in teaching and other public and private sector organizations. The Doctor of Philosophy (PhD) in Mathematics program is a minimum three years program comprised of eight regular semesters. Students have to be admitted to PhD in Mathematics program as per the Higher Education Commission (HEC) of Pakistan Criteria. This program will advance the understanding of the students on key mathematical issues both theoretically and empirically. Moreover, this program will prepare the students to conduct research on the key mathematical challenges being faced by institutions, organizations and industries at the regional, national and international levels. Degree of Doctor of Philosophy (PhD) in Mathematics program is designed to equip the students with the advanced knowledge of mathematical methods and procedures so that they could understand that how mathematical theory works in basic and applied research. The students would be trained to identify the mathematical problems followed by suggesting solutions to handle such issues.

Admission Criteria

Title of the Degree: Doctor of Philosophy in Mathematics

Principles and Purposes:

The admission in PhD program will be given as per Kohsar University Murree rules and HEC Policy.

Completion of Prior Degree:

Before entry into PhD (Mathematics) program, the student shall have been awarded his or her MS/MPhil in Mathematics or equivalent degrees.

Minimum GPA Requirement:

For admission in PhD programs, a minimum CGPA of 3.0 (out of 4.0 in the semester system) or First Division (in the annual system) in his/her MS/MPhil degree is required, whether such degree obtained from Pakistani or any other foreign universities.

Testing Requirement:

All applicants into PhD program shall be required to take one of the following subject specific admissions tests or as per the University rules:

i. The Graduate Record Examination (GRE) test administered by approved Education Testing Services.
ii. A graduate admission test administered by the Education Testing Council; or
iii. With the permission of the HEC/KUM, an equivalent test developed by the University for Admissions into PhD program and it is mandatory to qualify with 70% marks.

Program Details
MS Mathematics
Semester-Wise Break Down

First Year					
First Semester			**Second Semester**		
S. No.	**Course Title**	**Credit hours**	**S. No.**	**Course Title**	**Credit hours**
1	Core-1	x-x-x	1	Optional-1	x-x-x
2	Core-2	x-x-x	2	Optional-2	x-x-x
3	Core-3	x-x-x	3	Optional-3	x-x-x
4	Core-4	x-x-x	4	Optional-4	x-x-x
Total		**12**			**12**

Second Year	
MS Thesis (MTH-799)	6 Credit Hours

Program Details
Ph.D. Mathematics
Semester-Wise Break Down

Semester I			Semester II		
Course Code	**Course Title**	**Credit hours**	**S. No.**	**Course Title**	**Credit hours**
1	Elective-I	x-x-x	1	Elective-IV	x-x-x
2	Elective-II	x-x-x	2	Elective-V	x-x-x
3	Elective-III	x-x-x	3	Elective-VI	x-x-x
Total		**09**			**09**

Course Code	Course Title	Credit Hours
MTH-899	Thesis	30

***Rest of the things will be followed by HEC rules.**

List of Core Courses MS (Mathematics):

S. No	Course Code	Course Title	Credit Hour
1	MTH-601	Riemannian Geometry	3-0-3
2	MTH-602	Advanced Complex Analysis	3-0-3
3	MTH-603	Advanced Numerical Technique	2-1-3
4	MTH-604	Approximation Theory	3-0-3
5	MTH-605	Advanced Rings and Modules	3-0-3
6	MTH-606	Advanced Partial Differential Equations	3-0-3
7	MTH-607	Numerical Solution of Ordinary Differential Equations	2-1-3
8	MTH-608	Fixed Point Theory and Applications	3-0-3
9	MTH-609	Advanced Functional Analysis	3-0-3
10	MTH-610	Advanced Topics in Graph Theory	3-0-3
11	MTH-611	Applied Linear Algebra	3-0-3
12	MTH-612	Advanced Engineering Mathematics	3-0-3
13	MTH-613	Fuzzy Set Theory and its Applications	3-0-3
14	MTH-614	Advanced Real Analysis	3-0-3

***Ph.D. students may also select courses from the above list.**

List of Elective Courses for MS & Ph.D. Mathematics

S. No	Course Code	Course Title	Credit Hour
1	MTH-501	Numerical Methods	3-0-3
2	MTH-502	Linear Partial Differential Equations	3-0-3
3	MTH-503	Complex Variables and Transforms	3-0-3
4	MTH-504	Advanced Calculus	3-0-3
5	MTH-505	Nonlinear Dynamics and Waves	3-0-3
6	MTH-615	Advanced Group Theory	3-0-3
7	MTH-616	Hilbert Space Methods	3-0-3
8	MTH-617	Advanced Topology	3-0-3
9	MTH-618	Viscous-I	3-0-3
10	MTH-619	Number Theory	3-0-3
13	MTH-620	Commutative Algebra	3-0-3
12	MTH-621	Research Methodology	3-0-3
13	MTH-622	Mathematical Modeling in Physical Sciences	3-0-3
14	MTH-623	Advanced Mathematical Methods	3-0-3
15	MTH-624	Fuzzy Group Theory	3-0-3
16	MTH-625	Mathematical Inequalities and Applications	3-0-3
17	MTH-626	Theory of Semirings	3-0-3
18	MTH-627	Nilpotent and Soluble Groups	3-0-3
19	MTH-701	Mathematical Techniques for Boundary Value Problem	3-0-3
20	MTH-702	Perturbation Methods	3-0-3
21	MTH-703	Banach Algebras and C^*-Algebra	3-0-3
22	MTH-704	Geometric Function Theory	3-0-3
23	MTH-705	Rough Set Theory and its Applications	3-0-3
23	MTH-706	Biomathematics	3-0-3
24	MTH-707	Advanced Fluid Mechanics	3-0-3
25	MTH-708	Advanced Ring Theory	3-0-3
26	MTH-709	Finite Element Method	3-0-3
27	MTH-710	Numerical Linear Algebra	3-0-3
28	MTH-711	Numerical Solution of Partial Differential Equations	3-0-3

29	MTH-712	Algebraic Number Theory	3-0-3
30	MTH-713	Fuzzy Logic and Algebra	3-0-3
31	MTH-714	Cryptography	3-0-3
32	MTH-715	Mathematical Methods for Signal Processing	3-0-3
33	MTH-716	Impulsive Differential Equations	3-0-3
34	MTH-717	Finite Element Methods	3-0-3
35	MTH-718	Directive Study	3-0-3
36	MTH-801	Non-Newtonian Fluid Mechanics	3-0-3
37	MTH-802	Computational Fluid Dynamics	3-0-3
38	MTH-803	Advanced Integral Equations	3-0-3
39	MTH-804	Numerical Methods for Partial Differential Equations	3-0-3
40	MTH-805	Stochastic Differential Equations and its Application	3-0-3
41	MTH-806	Fluid Mechanics and Heat Transfer	3-0-3
42	MTH-807	Theory of Majorization	3-0-3
43	MTH-808	Nonlinear Analysis and Its Applications	3-0-3
44	MTH-809	Advanced Mathematical Physics	3-0-3
45	MTH-810	Lebesgue Spaces with Variable Exponent	3-0-3
46	MTH-811	Applied Dimensional Analysis and Modeling	3-0-3
47	MTH-812	Dynamical Systems and Control Theory	3-0-3
48	MTH-813	Advanced Dynamics	3-0-3
49	MTH-814	Statistical Methods for Data Science	3-0-3
50	MTH-815	Nonlinear Functional Analysis	3-0-3
51	MTH-816	Optimization Theory	3-0-3
52	MTH-817	Special Topics	3-0-3

*****Bridging courses of 500 level can be offered in zero semester for those students who are from other disciplines i.e., Physics, Computer Science etc.

*****The MS Mathematics students can study only 600 and 700 level courses in course work.

*****The PhD Mathematics Students can study 600, 700 and 800 level courses and a PhD student must select at least one course of 800 level in course work.

MTH-501 Numerical Methods

This course is an advanced introduction to applications and theory of numerical methods for solution of differential equations, especially of physically-arising partial differential equations, with emphasis on the fundamental ideas underlying various methods. Topics include finite differences, spectral methods, well-posedness and stability, boundary and nonlinear instabilities. The course assumes familiarity with basic (numerical) linear algebra and will involve a certain amount of programming in MATLABTM or any programming language of your choice.

Reference Books:

1. R. L. Burden, J. D. Fairs; An Introduction to Numerical Analysis, 1993.

2. G. D. Smith, Numerical Solutions of P.D.Es, 1999.

3. George Lindfield and John Penny, Numerical Methods Using MATLAB, Academic Press, 2019.

MTH-502 Linear Partial Differential Equations

This course covers the classical partial differential equations of applied mathematics: diffusion, Laplace/Poisson, and wave equations. It also includes methods and tools for solving these PDEs, such as separation of variables, Fourier series and transforms, eigenvalue problems, and Green's functions.

Reference Books:

1. A. K. Nandakumaran, P. S. Datti, artial Differential Equations: Classical Theory with a Modern Touch, Cambridge IISc Series, 2020.
2. Myint-U, Tyn, and Lokenath Debnath, Linear Partial Differential Equations for Scientists and Engineers, 2007.

MTH-503 Complex Variables and Transforms

Introduction to Complex Number System, Argand diagram, De Moivre's theorem and its Application Problem Solving Techniques, Complex and Analytical Functions,, Harmonic Function, Cauchy-Riemann Equations, Cauchy's theorem and Cauchy's Line Integral. Laplace transform, Laplace transform definition, Laplace transforms of elementary functions, Properties of Laplace transform, Periodic functions and their Laplace transforms, Inverse Laplace transform and its properties, Convolution theorem, Inverse Laplace transform by integral and partial fraction methods, Fourier series and Transform, Fourier transform definition, Fourier transforms of simple functions, Inverse Fourier transform.

Reference Books:

1. A Wilbur R. LePage, Complex Variables and the Laplace Transform for Engineers, Dover Publications; New edition, 2010.
2. T.K.V. Iyengar et al., Complex Variables and Transforms, S. Chand Publishing, 2022.

MTH-504 Advanced Calculus

Advanced topics of calculus include a review of vector and vector calculus, linear approximations of vector valued functions of several variables, the derivative matrix, real valued functions, multiple integrals, line integrals, surface integrals, and theorems of Green, and Stokes' divergence theorem.

Reference Books:

1. David V. Widde, Advanced Calculus, Dover Publications; 2nd edition, 1989.
2. Lynn Harold Loomis, Shlomo Zvi Sternberg, Advanced Calculus (Revised Edition), WSPC; Illustrated edition, 2014.

MATH-505 Nonlinear Dynamics and Waves

Introduction, Course organization, scope. Typical examples of nonlinearities in vibration and wave phenomena. Nonlinear Vibrations, Review of phase plane for one-d.o.f. systems, limit cycles. Perturbation techniques for weakly nonlinear systems. Nonlinear forced vibrations; jump phenomena, synchronization, superharmonic and subharmonic resonance. Extensions to multi-d.o.f. and continuous systems. Nonlinear Waves, Nonlinear dispersion relation and finite-amplitude periodic waves. Propagation of nonlinear pulses and the nonlinear Schrödinger equation. Long-crested waves and the Korteweg-de Vries equation. Nonlinear wave interactions. Forced nonlinear waves.

Reference Books:

1. Jüri Engelbrecht, Nonlinear Wave Dynamics Complexity and Simplicity, Springer, 1997.
2. Holm Altenbach et al., Nonlinear Wave Dynamics of Materials and Structures, Springer, 2020.

MTH-601 Riemannian Geometry

Differentiable Manifold, Smooth mappings, Partition of unity, Tangent space, smooth vector field, smooth tensor field, stokes theorem, Riemannain metric, examples of Riemannian metric, Affine connection, Geodesic, Exponential mapping, Geodesic convex neighborhood, Hopf Rinow theorem, Curvature tensor, Sectional curvature, Ricci curvature, Ricci identity, Jacobi filed, Cartan Ambrose theorem, Bonnet Myers theorem, Lie Groups.

Reference Books:

1. S. Kumaresan, Riemannian Geometry- Concepts, Examples and Techniques, Techno World, 2020.
2. Manfredo Perdigo do Carmo, Riemannian Geometry, Birkhauser Publisher, 1992.
3. John M. Lee, Riemannian Manifold: An Introduction to Curvature, Springer-Verlag, 1997.

MTH-602	**Advanced Complex Analysis**

Complex numbers, extended complex plan, Stereographic projection, Holomorphic functions as mappings, conformal mappings, Mobius transformations, branch of logarithmic functions, Riemann's Stleltigies integrals, power series representations of analytic functions, zeros of analytic functions, maximum modulus theorem, Cauchy theorem and integral formula on open subset of C, the homotopic version of Cauchy's theorem and simply connectedness, counting zeros, open mapping theorem, Goursats theorem, maximum modulus principle, Schwarz lemma, classification of singularities, Laurrent series, residues, meromorphic functions, Residue theorem, contour integration, argument principles and Rouches theorem.

Reference Books:

1. Bulboacă, Teodor, Differential subordinations and superordinations. New Results, Casa Cărţii de Ştiinţă Publishing House, Cluj-Napoca, 2005.
2. Churchill, R.V., Brown, J.W., Complex Variables and Applications 9 edition, McGraw Hill, New York, 2014.
3. Mocanu, Petru - Kohr, Gabriela: Special chapters of complex analysis, Cluj-Napoca, Cluj University Press, 2005.

MTH-603	**Advanced Numerical Technique**

Approximation of derivatives through Taylor series, truncation error, order of convergence, parabolic Partial Differential Equations, Explicit methods, Implicit methods, Numerical solutions of elliptic, parabolic and hyperbolic PDEs, Numerical solution to System of linear and nonlinear equations, Condition number and spectral properties of a matrix, Newton and fixed point methods, applications in MATLAB.

Reference Books:

1. Klaus A. Hoffmann, Computational Fluid Dynamics, Vol-II 4th Edition,2000.
2. Richard L. Burden and J. Dougles Faires, Numerical Analysis, 9th Edition, 2010.
3. John H. Mathews and Kurtis D. Fink Faires, Numerical Methods Unising MATLAB, 4th Edition, 2004.

MTH-604 Approximation Theory

Introduction to Legendre, Bernstein, Jacobi, Lagurre polynomials and their recurrence relations Chybshive, Bernoulli polynomials and their properties and recurrence relations. The Shifted Jacobi and The Shifted Legendre polynomials, the Hermit polynomials. Approximations of functions in terms of the above polynomials, The convergence analysis of the approximation of the above polynomials. The iterative technique and contraction mapping. Applications of above polynomials to the approximate solutions of differential and integral equations.

Reference Books:

1. Henryk G. and Jose' L. P,On the Approximation Properties of Bernstein polynomials via Probabilistic tools, Boletin de la Asociacion Matematica Venezolana.,2003 .
2. De Villiers, J. M, Mathematics of approximation, Atlantic Press, Amsterdam 2012.

MTH-605 Advanced Rings and Modules

Rings, ideals, ring homomorphism, quotient rings, properties of ideals, rings of quotients and localization, The Chinese Remainder Theorem, Euclidean domains, Principals ideals domains, unique factorization domains, rings of polynomials and formal power series, factorization in polynomial rings, Modules, Quotient modules, module homomorphisms, direct sum of modules, free modules, tensor product, exact sequences, Projective and injective modules, modules over principle ideal domains.

Reference Books:

1. David S. Dummit, Richard M. Foote, Abstract Algebra 3rd Edition, John Willey & Sons, 2004.
2. Thomas Hungerford, Abstract Algebra, Springer-Verlag GTM 73, 2003.
3. Pierre Antonie Grillet, Abstract Algebra, GTM 242, 2nd edition, Springer-Verlag, 2007.

MATH-606 Advanced Partial Differential Equations

Classification of partial differential equation, Canonical form, Laplace, Wave and Diffusion Equations, Partial differential Equation with at least three independent variables; Non-homogeneous problems; time independent problems; Infinite domain problems; time dependent problems; Wave equation and the method of characteristics, Eigen-function expansion method.

Reference Books:

1. Mrudul Y. Jani and Nita H. Shah, Partial Differential Equations: An Introduction, CRCpress, 2020.
2. Yehuda Pinchover and Jacob Rubinstein, An introduction to Partial DifferentialEquations, Cambridge, 2005.
3. Jürgen Jost, Partial Differential Equations, Springer. 2013.

MATH-607 Numerical Solutions of Ordinary Differential Equations

Initial value problems for ordinary differential equations, elementary theorem of IVP Higher-order Taylor method, Rungee-Kutta method Multi-step method (Rungee-Kutta'-Felberg Method) Higher order and system of DE. Development of computational code based on the above algorithm Boundary value problem of ODE. The linear shooting method shooting method for non-linear problems method of parametric differentiation Finite difference method for linear problem Finite difference method for non-linear method Reyleigh-Ritz method development of computational code based on the above algorithm.

Reference Books:

1. Le xie Fox, David Francis Mayers, Numerical Solution of Ordinary Differential Equations by,Chapman and Hall, 1997.
2. Suman Kumar Tumuluri, A First Course in Ordinary Differential Equations, Chapman andHall/CRC, 2021.
3. Alan Garfinkel, Jane Shevtsov, Yina Guo, Modeling Life: The Mathematics of BiologicalSystems, Springer, 2017.

MTH-608 Fixed Point Theory and Applications

Basic concepts: metric spaces, complete metric spaces, Definition and examples of fixed points, Common fixed points, Picards iteration, Banachs contraction principle, Edelstien's Fixed point theorem, Kannan and Chatterjee fixed point theorems, non-expansive mappings and related fixed point theorems. Contractive maps, Multi-valued mappings and related fixed point theorems. Fixed point results in complete metric spaces. Digital metric spaces and related fixed point result using various contraction conditions, Applications of fixed point theory.

Reference Books:

1. Andrzej Granas, James Dugundji, Fixed Point Theory, Springer, 2003.
2. Yeox Je Cho ,Jong Kyu Kim ,Shin Min Kang Fixed Point Theory and Applicationsby. NovaPublishers, 2007
3. Vasixe I .Istratescu ,Vasixe I Istră tescu, Fixed Point Theory: An Introduction, Springer, 2001.

MTH-609 Advanced Functional Analysis

A quick review of basic functional analysis, Compact linear operator on normed spaces, spectral of compact linear operators, operator equations involving compact linear operators, Spectral properties of bounded self adjoint operators, positive operators, projection operators, spectral family, spectral family of a bounded self adjoint operators, unbounded linear operators in Hilbertspaces, Hilbert adjoint operators, symmetric and self adjoint operators, multiplication and differentiation operators, Application of unbounded operators.

Reference Books:

1. Kreyzig, E., Introductory Functional Analysis with Applications, John Wiley and Sons, 1989.
2. Rudin, W., Functional analysis. International series in pure and applied mathematics, Academic Press, 1991.
3. Morosanu, G., Functional analysis for the applied sciences, Springer, 2019.
4. James C. Robinson, An Introduction to Functional Analysis, Cambridge University Press,2020.

MTH-610 Advanced Topic in Graph Theory

Graph Isomorphism andautomorphisms, Labelled graphs, Graphs arising from other structures, types of Graphs, Decomposition and coverings, Edge cuts and bonds, even sub-graphs, Graph reconstruction, Walks and connection, Cut edges,Connection to diagraphs, Cycle double covers, Forests and Trees, Spanning Tree, Calay's formula, Fundamental Cycles and Bonds, Co-tree,

Trees and Distance.Applications of Tree, Cut vertices, Separations and Blocks,Ear Decompositions, Strong Orientations, Directed Ear Decompositions, Even Cycles Decompositions, Vertex Connectivity, Fan Lemma, EdgeConnectivity, Three-connected graphs, Sub- modularity, Determining, Chordal graphs, Simplicial vertices, Plane and Planar graphs, Duality, Euler's formula, Bridges, Kuratowski's theorem, Chromatic numbers, Critical graphs, Girth and chromatic number, Perfect graphs, List colorings', The adjacency polynomial, Chromatic polynomial. Applications of vertex colorings to various problems. Edge colouring number, Vizing's theorem, Snarks, Covering by perfect matching, List edge coloring, Applications of edge colorings. Applications to various selected problems. Hamiltonian and non-Hamiltonian graphs, Non-Hamiltonian planar.

Reference Books:

1. Bondy, A., and Murthy,U. S. R., Graph Theory, Springer,2008.
2. Gross, J and Yellen, J., Graph Theory and its Applications, CRC Press London, 2005.
3. Wilson, R. J., Introduction to Graph Theory, Prentice Hall Wesley, 1996.

MTH-611 Applied Linear Algebra

Introduction, geometry of linear equations, Gaussian elimination, Vector spaces, homogeneous and non-homogeneous systems, linear dependence and independence, basis, dimensions, linear transformations, orthogonal vectors and subspaces, projections, orthogonal basis and Gram-Schmidt process, determinants and its applications, Eigen values and Eigen vectors, their applications in diagonalization and solutions of difference/differential equations, positive definite matrices, Inner product spaces, Applications of inner product spaces.

Reference Books:

1. David C. Lay, Lineaar Algebra and its applications, 4th edition, Pearson education, 2013.
2. Peter J. Olver, Chehrzad Shakiban, Applied Linear Algebra, Prentice-Hall, 2006.

MTH-612 Advanced Engineering Mathematics

Analytic Functions, applications to the problems of potential flow, Harmonic functions, Harmonic conjugates, Milne's method, Complex integration, sequences and series, uniform convergence, power series, Hadamard's formula for the radius of convergence, Taylor and Laurent series, zeros and poles of a function, meromorphic function, the residue at a singularity, Residue theorem, the argument principle and Rouche's theorem, contour integration and its applications to evaluation of a real integral, integration through a branch cut, conformal mapping, application to potential theory, review of unilateral and bilateral Z-transforms and their properties, application of calculus

of residues for the inversion formula of Z- transforms and Laplace transforms, review of Fourier integrals and Fourier transforms, Finite Fourier transforms, discrete Fourier transforms and applications, basic concepts of probability, Bayes theorem, probability networks, discrete and continuous probability distribution, joint distribution, correlation coefficient, applications to problems of reliability, queueing theory, service time for a customer in a facility and life testing, testing of hypotheses. This course has tremendous applications in diverse fields of Engineering and Sciences such as Signal processing, Potential theory, Bending of beams etc.

Reference Books:

3. Kreyszig, E., "Advanced Engineering Mathematics", Wiley, New York, 2011.
4. Ain, R.K. and Iyenger, S.R.K., Advanced Engineering Mathematics, 2nd Edition, Narosa Publishing House, 2008.

MTH-613 Fuzzy Set Theory and its Applications

Concept of Fuzzy set, Constructing Fuzzy Sets, Standard Operations on Fuzzy Set, Fuzzy numbers, Triangular Fuzzy Number, Trapezoidal Fuzzy Number, Decomposition Theorem, Extension Principle, Fuzzy arithmetic, Fuzzy relations and their characteristics, Applications in operation research, data mining and decision analysis.

Reference Books:

1. Wang et al. Advances in Fuzzy systems Nonlinear Integrals and their applications in data mining, Singapore: World Scientific Publishing Company, 2010.
2. Kwang H. Lee, First Course on Fuzzy Theory and Applications, Springer, 2005.

MTH-614 Advanced Real Analysis

General measure spaces, Signed Measure, Hahn and Jordan Decomposition theorems, Caratheodory Measure and outer measure. Integration over General Measure: Measurable function, integration of general measurable function and their properties. The Radon-Nikodym theorem, Radon measure on locally compact spaces and Riesz' representation theorem. Applications to Fourier analysis and probability theory: Heisenberg's inequality, the Prime Number Theorem, ergodic theory. Hansdorff measures.

Reference Books:

1H. Royden, P. Fitzpatrick, Real Analysis, 4th Edition, Prentice Hall, 2010.

2. Robert G. Bartle, Donald R. Sherbert, Introduction to Real Analysis, 4th Edition, Wiley sons.2018.
3. Liviu I Nicolaescu, Introduction to Real Analysis, WSPC, 2019.
4. Jay Cummings, Real Analysis: A Long-Form Mathematics Textbook, Create Space Independent Publishing Platform, 2018.

MTH-615 Advanced Group Theory

Group operations, orbit and stabilizer, p-groups, Sylow groups, Sylow's Theorem, nilpotent groups, Free groups, Action on a group, group presentations, isomorphism theorems, Jorder-Hölder Theorem, Solvable groups, definition, examples and related results, The Frattini subgroups, definition, examples and related results, Cohomology groups. Group algebras and representation modules. Fourier analysis on finite groups, representations of Lie groups/matrix groups.

Reference Books:

1. W. R. Scott, Group Theory Dover Publications Inc.; New edition, 2003.
2. J. B. Fraleigh and Neal E. Brand, A First Course In Abstract Algebra by, 8th Edition, Pearson Education, 2020.

MTH-616 Hilbert Space Methods

Vector spaces, Normed Spaces, Inner Product spaces, Completeness, Hilbert Spaces, Linear and bounded operators, Orthoprojections, Isometric and unitary operators, Continuous functions of self-adjoint operators, Coercivity, Elliptic forms, Regularity, closed operators, adjoint and eigenfunction expansions, Spectral Integrals, Differential Operatorsin $L^2(a, b)$.

Reference Books:

1. Karl E. Gustafson, Introduction to Partial Differential Equations and Hilbert Space Methods 1997.
2. R. A. Kennedy, P. Sadeghi, Hilbert Space Methods in Signal Processing, 2013.

MTH-617 Advanced Topology

Semi-open sets, semi-closed sets and their characterizations; semi-open, semi-closed mappings,almost closed and almost open mappings, s-continuous, s-open and s-closed functions and their properties. Semi-closure, semi-interior and their properties, weekly continuous and almost continuous mappings, semi-weekly continuous mappings and s-connectedness. Stronglycontinuous mapping and connectedness s-regular spaces, semi-Ti spaces, i=1, 2, 3 and their properties. Semi-regular, almost regular, almost completely regular, semi-normal, almost normal spaces and their properties, vector bundles, topological and/or

algebraic K-theory, Morse theory, bordism theory, spectra and stable homotopy theory, homotopical and higher algebra.

Reference Books:

1. Stephen Gaal, Point Set Topology, Dover Publications Inc.; 2009th edition, 2009.
2. George F .Simmons, Topology and modern analysis by. Krieger Publishing Company; 2003.
3. Miguel Arajo, Pedro Sacramento, Topology in Condensed Matter: An Introduction, WorldScientific Pub Co Inc. 2021.
4. Tai-Danae Bradley, Tyler Bryson, John Terilla, Topology: A Categorical Approach, The MIT Press, 2020.

MTH-618 Viscous-I

Convection, stability, boundary layers, parameterized convection, Rotating flows, atmosphere and oceans, Waves, geostrophy, boilers, condensers, fluidised beds. Flow régimes. Homogeneous, drift-flux, two-fluid models. Ill-posedness, waves, density wave oscillations. Coatings and foams. Gravity flows, Droplet dynamics, contact lines, Drying and wetting, Foam drainage.

Reference Books:

1. Frank M. White, Viscous Fluid Flow, Second Edition, McGRAW-HILL, Inc, 1991.
2. G.K. Batchelor, An introduction to fluid dynamics,Cambridge University Press, 2000.

MTH-619 Number Theory

Topical Outline of the Course Content:

Properties of Integers, such as divisibility, congruences and prime numbers. More advanced topics include encryption, quadratic reciprocity and Diophantine approximation. Algebraic and transcendental numbers, algebraic integers, quadratics fields, quadratic forms, ideal theory in quadratic fields, continued fractions and approximations, partitions, number-theoretic functions, number theoretic equations.

Reference Books:

1. D. M. Burton, Elementary Number Theory, McGraw-Hill, 2007.

2. K. H. Rosen, Elementary Number Theory and its applications, 5^{th} edition, Addison-Wesley, 2005.

MTH-620 Commutative Algebra

Basics of ring theory, Field extensions, algebraic extensions, Galois extensions, Algebraic sets, Algebraic varieties, Zarisky Topology, Nullstellensatz, Primary submodules and ideals, primary decompositions, associated primes, support of a module, integral elements, integral closures, Normalization, localization rings and modules, Extension theorems, valuation rings, Discrete Valuation Rings (D.V.R.), Graded rings and modules, completion of a module, The Krull intersection theorem, Regular local rings.

Reference Books:

1. M.F. Atiyah, I.G. Macdonald; Introduction to commutative algebra, Addison-Wesley Publishing Co., 1969.
2. D. Eisenbud; Commutative Algebra with a view toward Algebraic Geometry, Springer-Verlag GTM 150, 1995.
3. Pierre Antoine Grillet, Abstract Algebra, GTM 242 second edition, Springer-Verlag, 2007.

MTH-621 Research Methodology

Meaning and objectives of research, the research process, criteria of good research, research ethics, selecting a method of data collection, collecting data using attitudinal scales, establish the validity and reliability of a research instrument, selecting a sample, considering ethical issues in data collection, processing data, displaying data, plagiarism policy, use of digital library, review of research articles, managing the literature, guidelines for synopsis, components of a synopsis, size and format, submission, tools for writing scientific reports, plotting tools, bibliography and its styles, poster preparation and presentation guidelines.

Reference Books:

1. Alexander Zaetsky, Research Methodology and techniques in Mathematics, Delve Publishing

LLC, 2015.

2. Kaifeng Yang, Gerald , Miller, Lucienne T. M, Research Methods, CRC Press, 2007
3. J Blessing & Amaresh, Chakrabarti, Research Methodology, Springer, 2009.
4. Joseph Herbert Abramson, Z. H. Abramson Research Methods, Wiley, 2008.

MTH-622 Mathematical Modeling in Physical Sciences

Qualitative modeling with functions, Modeling with dimensional analysis, Modeling with Difference equations :(a) Overview of basic concepts concerning matrices, eigenvalues and eigenvectors; (b) Fixed points, stability and iterative processes;(c) Applications to population growth. Modeling with Ordinary Differential Equations, Overview of basic concepts in ODE and stability of solutions: existence and uniqueness for 1st order IVPs, Picard iteration, numerical methods, higher order IVPs, Empirical Modeling with Data Fitting, Error function, least squares method;(b) Fitting data with polynomials and splines, Modeling with Partial Differential Equations:(a) Overview of the key properties of some particular kinds of PDEs, Advection, diffusion, advection-diffusion, (b) Separation of variables, equilibrium solutions, stability and linear stability, (c) Travelling waves, spatially periodic solutions (patterns), (d) Some applications: stripes on the skin of the Marine Angelfish, Analysis of temperature from the Greenland Ice Sheet, Presentations related to the area.

Reference Books:

1. Giordano et al., A First Course in Mathematical Modeling, Cengage Learning; 5th edition, 2013.
2. Haddon, M., Modeling and Quantitative Methods in Fisheries, A CRC, 2014.
3. Boss,L. M., Mathematical Methods for Physical Sciences, John Wiley & Sons, Inc, 2006.

MTH-623 Advance Mathematical Methods

Fourier series expansion, general properties, Uses of Fourier series, derivation of Reimann zeta function, Integral Transforms. Sine and Cosine Transforms, Fourier Transform of Derivatives, Convolution theorem. Parseval's relation, Momentum representation, examples, Laplace Transforms, Laplace Inverse Transform, Laplace Transform of Derivatives. Other properties of Laplace Transform, Convolution Theorem, Integration of Transforms, examples, Introduction to Fred holmand volterra equations, examples, Transformation of a Differential Equation into an Integral Equation example of linear oscillator equation, Generating Functions, examples, Neumann Series, Separable (Degenerate) Kernels. Hilbert–Schmidt Theory, Bessel functions of first kind and its generating function. Recurrence relations of Bessel function, derivation of Bessel's differential equation, Integral representation of Bessel functions, Orthogonality and normalization of Bessel functions. Neumann function-Bessel functions of second kind, Wronskian Formulas, Hankel functions, Modified Bessel functions, Asymptotic expansions. Spherical Bessel functions, Generating function of Legendre Functions, Recurrence relations and special Properties of Legendre Functions, Orthogonality, Associated Legendre functions. Euler, Tyler methods, Heun;s methods and applications.

Reference Books:

1. Kelley, J. Graduate Mathematical Physics. Wiley, 2006.
2. Arfken, G. B. Weber, H. J. Harris, F. E. Mathematical Methods for Physicists, 7th edition, Elsevier Academic Press, 2013.
3. Advanced Engineering Mathematics, 10th edition, Erwing Keryszig, John Wiley & Sons New York, 2011.

MTH-624 Fuzzy Group Theory

Fuzzy subsets and fuzzy subgroups. Fuzzy Caley's theorem and fuzzy Lagrange's theorem. Nilpotent, commutators, and solvable fuzzy subgroups. Characterization of certain groups and fuzzy subgroups. Fuzzy subgroup of abelian groups, Direct products of fuzzy subgroups and fuzzy cyclic subgroups. Equivalence of fuzzy subgroups of finite abelian groups. Lattices of fuzzy subgroups.

Reference Books:

1. J. N. Mordeson, R. B. Kiran and A. Rosenfeld, Fuzzy Group Theory, Published by Springer, 2005.
2. J. George, Fuzzy Sets and Fuzzy Logic: Theory and Applications 1st Edition, 1995.

MTH-622 Mathematical Inequalities and Applications

Jensen's and Jensen-Steffensen's inequalities, Hermite-Hadamard's, inequilities Slater's and some companion inequalities to the Jensen inequality, deduction of the Hölder, Cauchy Schwartz, AM-GM inequalities from Jensen's inequality, Young's, some determintal and matrix inequalities, Chebyshev's and Gruss inequalities, refinements, reverses, generalizations, multi-dimensional version and applications of the above mentioned inequalities.

Reference Books:

1. E. F. Beckenbach and R. Bellman, Inequalities, Springerverlag, Berlin, 1961.
2. Praveen A. et al., Advances in Mathematical Inequalities and Applications, Birkhäuser, 2019.
3. D. S. Mitrinović, Analytic Inequalities, Springer-Verlag, Berlin, 1970.

MTH-626 Theory of Semirings

Hemirings, Semirings: Definitions and examples of Hemirings and Semirings. Building new semirings from old. Complemented elements in semirings, ideals is semirings. Prime and semiprime ideals in semirings. factor semirings, Morphisms of semirings. Regular semirings, Semimodules over semirings, Morpisms of semimodules, Factor semimodules, Free projective and injective semimodules.

Reference Books:

1. J. S. Golan, The theory of Semirings and their applications in mathematics and theoretical computer science, Longman Scientific & Tehnical John Wiley & sons New York, 1992.
2. U. Hebisch and H. J. Weinert, Semirings algebraic theory and applications in computer science, World Singapore, New Jersey London Hong Kong, 1998.

MTH-627 Nilpotent and Soluble Groups

Normal and Subnormal Series, Abelian and Central Series, Direct Products, Finitely Generated Abelian Groups, Splitting Theorems, Soluble and Nilpotent Groups, Commutators Subgroup, Derived Series, The Lower and Upper Central Series, Characterization of Finite Nilpotent groups, Fitting Subgroup, Frattini Subgroup, Dedekind Groups, Supersoluble Groups, Soluble Groups with Minimal Condition. Subnormal Subgroups, Minimal Condition on Subnormal Subgroups, The Subnormal Socle, the Wielandt Subgroup and Wielandt Series, T-Groups, Power Automorphisms, Structure and Construction of Finite Soluble T-Groups.

Reference Books:

1. D.J.S. Robinson, A Course in the Theory of Groups, Graduate Textes in Mathematics 80, Springer, New York, 1982.
2. K. Doerk, T. Hawkes, Finite Soluble Groups, De Gruyter Expositions in Mathematics 4, Walter De Gruyter, Berlin, 2011.

MTH-701 Mathematical Techniques For Boundary Value Problems

Green's function method, Regular and singular perturbation techniques, method of strained co-ordinates, the method of matched asymptotic expansion, Homotopy perturbation method, Homotopy analysis method, Adomian decomposition method, Variation iterative method, Analytical solutions of various equations, Navier stokes equations.

Reference Books:

1. Nafeh, A.H., Perturbation Methods, John Wiley & Sons, 2000.
2. David Powers, Boundary Value Problems and Partial Differential Equations, Springer 2005.
3. Bashir Ahmad , Johnny L Henderson, Rodica Luca, Boundary Value Problems for Fractional Differential Equations and Systems, World Scientific Publishing Company, 2021.
4. Mustafa Avci, A Closer Look at Boundary Value Problems, Nova Science Pub., 2020.

MTH-702 Perturbation Methods

Parameter perturbations, Co-ordinate perturbations, Order symbols and gauge functions, Asymptotic series and expansions, Asymptotic expansion of integrals, Integration by parts, Laplace's method and Watson's lemma, Method of stationary phase and the method of steepest descent. Straightforward expansions and source of non-uniformity, the Duffing equation, small Reynolds number flow past a sphere, small parameter multiplying the highest derivative.The method of strained co-ordinates, Lindstedt-Poincare method, renormalization method.Variation of parameters and method of averaging, examples. Method of multiple scales with examples.

Reference Books:

1. Nafeh, A.H., Perturbation Methods, John Wiley & Sons, 2000.
2. Nafeh, A.H., Introduction to perturbation Techniques, John Wiley & Sons, 1993.
3. Reza N. Jazar, Perturbation Methods in Science and Engineering, Springer, 2021.

MTH-703 Banach algebras and C*-algebras

A brief review of Hilbert space, Basic definitions of linear operators, mostly on Hilbert space, Compact operators on Hilbert Spaces, Definition of a Banach algebra, Spectrum, spectral radius, holomorphic functional calculus, The weak and weak* topologies, Alaoglu's Theorem, Krein-Milman Theorem, Maximal ideal space of a commutative Banach algebra, Gelfand transform, Definition of a C*-algebra, Foundations of the general theory of C*-algebras: continuous functional calculus, positive elements, ideals, quotients, approximate identities, States and representations: GNS construction. Existence of irreducible representations

Reference Books:

1. Rudin W, Functional Analysis, McGraw Hill, New York, 1973.
2. E. Kreyszig, Introductory Functional Analysis with Applications, Wiley classics library, John Wiley, 2007.
3. W. Rudin, Functional Analysis, McGraw-Hill, N.Y., 2006.

MTH-704 Geometric Function Theory

Riemann mapping theorem, conformal mappings and their properties, univalent functions and their subclasses, some area theorems, elementary bounds for the coefficients, some theorems on power series, coefficients for the area theorem, implications of the bound on the second coefficient, functions with positive real part, Herglotz formula, the technique of dominancepower series, subordination, the Noshiro-Warschawski theorem, some basic properties of univalent and

multivalent functions, special classes of univalent functions, Convex and Starlike functions, sharp bound for the analytic functions, Convex and Starlike functions of order alpha, Alpha convex functions, Alpha spiral like functions.

Reference Books:

1. Steven George Krantz, Geometric Function Theory: Explorations in Complex Analysis , 2006
2. Daniel Glossman-Mitnik, Density Functional Theory, Intechopen, 2019
3. Carl Hanson Fitz Gerald, Topics in geometric function theory by, Stanford University, 2001.

MTH-705 Rough Set Theory and its Application

Introduction the course, Knowledge base, Approximations and rough sets, membership function, operations on rough sets, reduct and core of knowledge, dependencies in knowledge, knowledge representation, Applications of rough sets in Algebra, generalization of rough sets, applications of rough sets in decision making and machine learning.

Reference Books:

1. Pawlak and Zdzistaw, Roughs Sets: Theoretical aspects reasoning about data, Vol. 9, Springer Science & Business Media, 2012.
2. Lech Polkowski, Rough Sets: Mathematical Foundation, Physica-Verlag HD, 2002.

MTH-706 Biomathematics

Students will learn the development, analysis, and interpretation of biomathematical models based on discrete-time and continuous-time models. Applications may include examples from population biology, ecology, infectious diseases, microbiology, and genetics. Setting up Mathematical models. Analysis of discrete time dynamical systems and their application to biology, stability analysis, data fitting to discrete models, Linear ODE, system of ODE, non-linear ODEs.

Reference Books:

1. Shonkwiler, R., and Herold, J., Mathematical Biology, Springer, Heidelberg, 2009.
2. Avner Friedman, Mathematical Biology: Modeling and Analysis, AMS, 2018
3. Britton, Nicholas Ferris. Essential mathematical biology, Springer, 2003

MTH-707 Advanced Fluid Mechanics

This course is a survey of principal concepts and methods of fluid dynamics. Topics include mass conservation, momentum, and energy equations for continua; Navier-Stokes equation for viscous flows; similarity and dimensional analysis; lubrication theory; boundary layers and separation; circulation and vorticity theorems; potential flow; introduction to turbulence; lift and drag; surface tension and surface tension driven flows.

Reference Books:

1. Batchelor, G.K., An Introduction to Fluid Dynamics, Cambidge University Press, 1969.
2. C. Kleinstreuer, Modern Fluid Dynamics, CRC Press, 2018.
3. Peiqing Liu, A General Theory of Fluid Mechanics, Springer; 2021.
4. Gregory Falkovich, Fluid Mechanics 2nd Edition**,** Cambridge University Press, 2018.

MTH-708 Advanced Ring Theory

Radical classes, semi simple classes, the upper radical, semi simple images, the lower radical, hereditariness of the lower radical class and the upper radical class. Partitions of simple rings, Minimal left ideals, Wedderburn-Artin structure theorem, The Brown-McCoy radical, The Jacobson radical, Connections among radical classes, Homomorphically closed semi simpleclasses.

Reference Books:

1. Wiegandt, R., Radical and Semi simple classes of Rings, Queen's papers in Pure and AppliedMathematics No.37, Queen's University, Kingston, Ontario, 1974.
2. Grillet, P. A., Abstract Algebra Springer, 2007.
3. Donald S. Passman ,A Course in Ring Theory, Chelsea Pub Co, 2004.

MTH-709 Finite Element Method

Introduction to Sobolev spaces, Ritz-Galerkin approximation of Poisson's equation, weak formof Poisson's equation, variational form of Poisson's equation, Ritz-Galerkin approximation of Poisson's equation with hat functions, elliptic bilinear form, elliptic variational form, Ritz-Galerkin approximation of an elliptic variational problem, construction of FE basis, properties of basis function, basis function of multidimensional space, linear independence of basis function, basis function on uniform grid, condition number of Galerkin matrix, uniform Lagrange

polynomial, extension of basis function, coefficients of extended basis, weight functions, R-functions, partial weight function, WEB-splines, stability and approximation with WEB-spline, Ritz-Galerkin system, applications of WEB-approximation.

Reference Books*:*

1. Andersson, L. E., Introduction to the Mathematics of Subdivision Surfaces SIAM, 2010.
2. Quarteroni, A., Numerical Models for Differential Problems, Springer, 2009.
3. Bathe, K. J., Finite Element Method by, John Wiley & Sons, 2007.
4. Horst Werkle, Finite Elements in Structural Analysis, Springer, 2021.
5. Randy Shih, Introduction to Finite Element Analysis Using SOLIDWORKS Simulation, SDC, *2021*.

MTH-710 Numerical Linear Algebra

Fundamentals: matrix multiplication, orthogonal vectors, orthogonal matrices, norms: Gaussian elimination and its variant: cholesky decomposition, LU decomposition, pivoting strategies: sensitivity of linear systems: conditioning and stability; the least square problem and SVD; eigen values and eigenvectors III-posed ness &regularization.

Reference Books:

1. Lxoyd Nichoxas Trefethen ,David Bau .Numerical linear algebra by SIAM: Society for Industrial and Applied Mathematics; 1st edition, 1997.
2. Tom Lyche, Numerical Linear Algebra and Matrix Factorizations, Springer, 2020.
3. Charu C. Aggarwal, Linear Algebra and Optimization for Machine Learning: A Textbook, Springer, 2020.

MTH-711 Numerical Solutions of Partial Differential Equations

Parabolic equations: explicit finite difference approximation, crank nickelson implicit method: derivative boundary condition, The local truncation error; Stability, The necessary and sufficient condition for stability; Finite difference approximation in spherical polar coordinate Hyperbolic Equations Analytic solution of quasi-linear equations; Finite difference method on a rectangular mesh for first order equations Reduction of a first order equation to a system of ordinary differential equation; Second order quasi-linear hyperbolic equations; Finite difference method on

a rectangular mesh for second order equation; Simultaneous first order equations and Elliptic Equation, Systematic iterative methods for large linear systems; Methods for accelerating convergence; The Gauss Seidel iteration matrix.

Reference Books:

1. K . W. Morton, David Francis Mayers, Numerical solution of partial differential equations, Cambridge University Press, 2005.
2. Bert Hubbard, Numerical Solution of Partial Differential Equationsby. Academic Press, 2014.
3. Mrudul Y. Jani and Nita H. Shah, Partial Differential Equations: An Introduction, CRC press, 2020.

MTH-712 Algebraic Number Theory

Algebraic Numbers: Algebraic Numbers and Number Fields; Discriminant; Norms and Traces; Algebraic integers and Integral Bases; Factorization and Divisibility; Applications of UFD. Arthmetic's of Number Fields: Quadratic Dields; Cyclotomic fields; Units in Number rings. Ideals Theory: Properties of Ideals; PIDs and UFDs; Dedekind rings; Norms of ideals; Class group and Class Numbers of Quadratic Fields. Valuations: Definitions and First properties of valuations; Valuation rings; DVRs; p-adic valuation.

Reference Books:

1. Richard A. Molin, Algebraic Number Theory, Chapman & Hall, Washington D. C., 2005.
2. Akash Singha Roy and Paul Pollack, Steps into Analytic Number Theory: A Problem-Based Introduction, Springer, 2021.
3. Robin Wilson, Number Theory: A Very Short Introduction, Oxford press, 2021.

MTH-713 Fuzzy Logic and Algebra

Introduction, The Concept of Fuzziness: Examples; Mathematical Modeling; Operations of fuzzy sets; Fuzziness as uncertainty, Algebra of Fuzzy Sets: Boolean Algebra and lattices; Equivalence relations and partitions; Composing mappings; Alpha-cuts; Images of alpha-level sets; Operations on fuzzy sets. Fuzzy Relations: Definition and examples; Binary Fuzzy relations Operations on Fuzzy relations; fuzzy partitions. Fuzzy Semigroups: Fuzzy ideals of Semigroups; Fuzzy quasi-ideals; Fuzzy bi-ideals of Semigroups; Characterization of different classes of Semigroups by the properties of their fuzzy ideals fuzzy quasi-ideals and fuzzy bi-ideals. Fuzzy Rings: Fuzzy ideals of rings; Prime; semi prime fuzzy ideals; Characterization of rings using the properties of fuzzy ideals.

Reference Books:

1. Nguyen, H. T., and Walker, E. A., A First Course in Fuzzy Logic, Chapman and Hall/CRC 1999.
2. Ganesh, M., Introduction to Fuzzy Sets and Fuzzy Logic,Prentice-Hall of India, 2006.
3. Mordeson, J. N., Malik, D. S.,and Kuroki, N., Fuzzy Semigroups, Springer-Verlage, 2003.

MTH-714 Cryptography

Overview of Cryptography. What is a cipher? One time pad and stream ciphers, block ciphers, Message integrity: definition and applications, Collision resistant hashing, authenticated encryption: security against active attacks, Arithmetic modulo primes and finite cyclic groups, Cryptography using finite cyclic groups, Public key encryption using a trapdoor function, Digital signatures: definitions and applications, Authenticated key exchange and SSL/TLS session setup, Cryptography in the age of quantum computers, The crypto wars: should we have end-to-end encryption?

Reference Books:

1. Paulpete Cercenia, Dan Boneh Boneh, Victor Shoup , A Graduate Course in Applied Cryptography: English Paperback – Large Print, 2022
2. Jonathan Katz and Yehuda Lindell; Introduction to Modern Cryptography: Third Edition (Chapman & Hall/CRC Cryptography and Network Security Series), Chapman and Hall/CRC; 3rd edition, 2020.

MTH-715 Mathematical Methods for Signal Processing

Introduction to signal processing, Signal formation and sampling in MRI and FMRI, Representation of a signal in the frequency domain (Fourier methods), Compression (wavelets), Filter operations for attenuation of a signal, Extracting information from a signal without a model (PCA,ICA), De-noising, Applications in speech and biomedical signal processing, Applications in learning how the brain functions (FMRI).

Reference Books:

1. Mathematical Methods and Algorithms for Signal Processing, Todd K. Moon, Wyn C. Stirling, Prentice Hall, 2008.

MTH-716 Impulsive Differential Equations

Introduction to impulsive differential equations. Investigation of initial and boundary value problems of impulsive differential equations, their existence theory and stability analysis, Study of nonlocal boundary value problems of impulsive differential equations. Investigation of implicit type impulsive differential equations and their applications.Iterative techniques for impulsive differential equations.

Reference Books:

1. V Lakshmikantham , D D , P S Simeonov, Theory of Impulsive Differential Equations,Series in Modern Applied Mathematics: Volume 6, 1989.
2. Mykola Perestyuk, Impulsive differential equations, Institute of Mathematics, Academy of Sciences of the Czech Republic, 2011.
3. Yong Zhou, Basic theory of fractional differential equations , World Scientifc Publishing Co. Pte. Ltd, 2014.

MTH-717	**Finite Element Methods**

History and derivation of the methods. The structure of finite element methods Applications. Solutions of some illustrative problems of PDEs and fluid problems by the FEM, Various types of finite element methods like generalization of the finite element method, conforming and non-conforming element method Smoothed Finite Element Methods, Spectral element methods, Meshfree methods .Discontinuous Galerkin method, Finite element limit analysis, Stretched grid method.

Reference Books:

1. Daryl L. Logan,. A first course in the finite element method, Cengage Learning, 2011.
2. P. Solin, K. Segeth, I. Dolezel: Higher-Order Finite Element Methods, Chapman & Hall/CRC, Press, 2003.

MTH-718	**Directive Study**

MTH-801 Non-Newtonian Fluid Mechanics

Classification of Non-Newtonian Fluids, Rheological formulae (Time-independent fluids, Thixotropic fluids and viscoelastic fluids), Variable viscosity fluids, Cross viscosity fluids, The deformation rate, Viscoelastic equation, Materials with short memories, Time dependent viscosity. The Rivlin-Ericksen fluid, Basic equations of motion in rheological models. The linear viscoelastic liquid, Couette flow, Poiseuille flows. The current semi-infinite field, Axial oscillatory tube flow, Angular oscillatory motion, Periodic transients, Basic equations in boundary layer theory, Orders of magnitude, Truncated solutions for viscoelastic flow, Similarity solutions, Turbulent boundary layers, Stability analysis.

Reference Books:.

1. Schowalter, W.R., Mechanics of Non-Newtonian fluids, New York, Pergamon Press 1978.
2. G. Falkovich, Fluid Mechanics, Cambridge press, 2018.
3. C. Kleinstreuer, Modern Fluid Dynamics, CRC Press, 2018.
4. P. Liu, A General Theory of Fluid Mechanics, Springer, 2021.

MTH-802 Computational Fluid Dynamics

Introduction to CFD, classification and mathematical behavior of partial differential equations, velocity pressure method for compressible flow, Non-iterative methods: method of superposition, method of Chasing, the adjoint operator method, methods of transformation; direct transformation, reduced physical parameters. Iterative methods: quasi linearization, the first upwind method, the Lax-wendroff method. Multigrid methods, Lagrange Multiplier method for BVP's, Spectral Method.

Reference Books:

1. Hoffmann, K. A., and Chiang, S. T., Computational Fluid Dynamics Vol I&II, EES, USA, 1993.
2. C. Kleinstreuer, Modern Fluid Dynamics, CRC Press, 2018.

MTH-803 Advanced Integral Equations

Introduction, classification of linear integral equations, Fredholm integral equations, homogeneous Fredholm equations, Volterra integral equations, Integro-differential equations, Fredholmintegro-differential equations, Volterraintegro-differential equations, Singular integral equations, Non-

linear integral equations. System of Voltera integral equations and system of Fredholm integral equations, System of singular integral equations, Nonlinear Voltera, Fredholm and singular integral equations with solution techniques, Applications of integral equations.

Reference Books:

1. Wazwaz, A.M., A first course in integral equations, World Scientific, 2015.
2. Harendra Singh, Hemen Dutta, Marcelo M. Cavalcanti, Topics in Integral and IntegroDifferential Equations: Theory and Applications, Springer; 2021.
3. D.C. Sharma, M. C. Goyal, Integral Equations, PHI Learning; 2017.

MTH-804 Numerical Methods Partial Differential Equations

Partial Differential Equations: Conservation Laws in Integral and Differential Form, One-Dimensional Burgers Equation, Convection, Characteristics for One-Dimensional Burgers Equation, Diffusion, Convection-Diffusion, Linear Elasticity Introduction to Finite Difference Methods: Finite Difference Approximations, Finite Difference Methods, Finite Difference Method Applied to 1-D Convection, Forward Time-Backward Space FTBS Analysis of Finite Difference Methods: Local Truncation Error for a Derivative Approximation, Truncation Error of Central Difference Approximation, Truncation Error for a PDE, Finite Difference Methods in Matrix Form, General Finite Difference Approximations, Boundary Conditions for Finite Differences, Introduction to Finite Volume Methods, Upwinding and the CFL Condition: Upwinding,The CFL Condition Eigenvalue Stability of Finite Difference Methods, Functional Approximation of the Solution.

Reference Books:

1. Numerical Methods for Partial Differential Equations Spring 2009.
2. David Francis Mayers and Keith William Morton, Numerical Solution of Partial Differential Equations, Cambridge University Press New York, NY, USA ,2005.

MTH-805 Stochastic Differential Equations and Its Applications

Riemann Integral and Stieltjes Integral, Normal (Gaussian) Distributions, Stochastic Process,Properties of BrownianMotion Paths, Markov Property of BrownianMotion, Noise, kinds of Noises, Additive and Multiplicative Noise, RandomWalk, Definition of Itˆo Integral, related theorems and lemmas, Itˆo Integral and Gaussian Processes, Definition of Stochastic Differential Equations, Solutions to Linear SDEs, Boundedness in Probability and Stability of Stochastic Processes, Defined by Differential Equations, Stationary and Periodic Solutions of Differential Equations, Markov Processes and Stochastic Differential Equations, Ergodic Properties of Solutions of Stochastic Equations, Stability of Stochastic Differential Equations, Systems of Linear Stochastic Equations, Stability in probability, Almost sure exponential stability, Moment exponential stability, Stochastic stabilization and destabilization, Noise independent /dependent of population sizes,

Stochastic delay Lotka–Volterra food chain

Reference Books:

1. Fima CKlebaner, Introduction to Stochastic Calculus with Applications, 2nd Edition, Imperial College Press57 Shelton StreetCovent GardenLondon WC2H 9HE, 2005
2. Rafail Khasminskii, Stochastic Stability of Differential Equations, 2nd Edition,Springer-Verlag Berlin Heidelberg 2012.
3. Xuerong Mao, Stochastic Differential Equations and Applications, Horwood PublishingChichester, UK Horwood Publishing Limited, 2nd Edition, 2007.

MTH-806	**Fluid Mechanics and Heat Transfer**

The course introduces fluid mechanics and heat transfer. The fluid mechanics section includes: fluid statics, the flow of liquids (incompressible flow) and gases (compressible flow) through tubing and various kinds of process apparatus. The flow of gases and liquids through packed and fluidized particle beds. Bernoulli´s equation. The heat transfer section includes heat transfer by conduction, convection and radiation in liquids, gases and solids. Dimensional analysis and CFD (Computational Fluid Dynamics) will be explained and used in both the fluid mechanics and heat transfer sections.

Reference Books:

1. John C. Tannehill et al., Computational Fluid Mechanics and Heat Transfer, CRC Press; 4th edition, 2020.
2. K.S.N. Raju, Fluid Mechanics, Heat Transfer, and Mass Transfer, Wiley, 2011.

MTH-807	**Theory of Majorization**

Basic theory of convex functions, weighted and unweighted majorization theorems applications of majorization theorem, Fuch's, Dragomir's, Shoshana- Pečarić's, Niezgoda's majorization results and relations between their results, generalization of majorization theorem for the class of n-convex functions by using Taylor's formula and Green function as well as Abel-Gontscharoff Polynomial, n-exponential convexity, n-exponential convexity for the functional obtained from the generalized majorization inequalities, some examples for exponential convexity results, Favard's and Berwald's inequalities and their generalizations.

Reference Books:

1. Albert W et al., Inequalities: Theory of Majorization and Its Applications, Springer, 2010.
2. J. Pečarić, F. Proschan and Y. L. Tong, Convex functions, Partial Orderings and Statistical Applications, Academic Press, New York, 1992.
3. A. W. Marshall, I. Olkin and B. C. Arnold, Inequalities: Theory of Majorization and Its Applications (Second Edition), Springer Series in Statistics, New York 2011.

MTH-808 Nonlinear Analysis & Its Applications

Banach and Hilbert spaces, contraction operators. Introduction to homotopy and its applications. Monotone iterative techniques of contraction operators, Fixed point theorem of Banach, Schaudar, Schaefer, Brower's and applications in nonlinear differential and integral equations,Degree theory and its applications,the Brouwer degree theory, The Schauder degree theory and topological degree theory.,The Picard-Lindelop theorem and its applications

Reference Books:

1. Martin Schechter, An Introduction to Nonlinear Analysis, University of California, Irvine, Cambridge University Press 2004.
2. Pavel Dr ´ abek, Jaroslav Milota, Lectures on Nonlinear Analysis, Czech Republic, Plzen-Prah,2004.
3. Collin Adams and R. Franzosa, Introduction to topology pure and applied , second edition 2012.

MTH-809 Advanced Mathematical Physics

Introduction to Vector spaces, inner product and Hilbert spaces, Rotations, transformations, & linear operators, Eigenvalues, Diagonalization, and Special Matrices with examples. Spectral properties of linear operator's spectrum and resolvent set. Radius of convergence of matrices, Normal modes examples related results, Curvilinear coordinates and parallel transport. Differential operators in curvilinear coordinates, Green's functions; self-adjoint differential equations; hermitian operators, Gram-Schmidt orthogonalization process; orthogonal polynomials; completeness of the Eigen functions, Bessel's inequality; Schwarz inequality; expansion of Green's functions; Green's functions in one dimension; Dirac delta function; gamma function; Bessel functions of the first kind, Legendre polynomials; associated Legendre polynomials; Spherical function, Hermite polynomials; Laguerre polynomials; associated Laguerre polynomials.

Reference Books:

1. Butkov, Mathematical Physics Addison Welley, 2008.
2. Courant & Hilbert, Methods of Mathematical Physics, Welley, 2007.
3. Arfken & Weber, Mathematical Methods for Physicists6th ed. Elsevier, 2011.

MTH-810 Lebesgue Spaces with Variable Exponent

Classical Lebesgue spaces, Lebesgue Space with variable exponent, Space of Homogenous type, History of variable exponent spaces, Elementary properties, Maximal Function, One-sided Maximal Function, Logrithmic Holder continuity, point wise estimates, the boundedness of the Maximal operators, the boundedness of Potential operators, Hardy-type Transforms, Week type estimates, Necessary Conditions for the boundedness, Weighted Lebesgue Space, One-sided Potentials,Logrithmic Holder continuity, point wise estimates, the boundedness of the Maximal operators, the boundedness of Potential operators, Hardy-type Transforms, Week type estimates, Necessary Conditions for the boundedness, Weighted Lebesgue Space, One-sided Potentials.

Reference Books:

1. Lars Diening et al., Spaces with variable exponents, Springer International Edition, 2010.
2. Ioseb Genebashvil et al., Weight Theory of Integral Transforms on Spaces of Homogenous Type, (Pitman Monographs& Surveys in Pure and Applied Mathematics), 1997.
3. Alexender Meskhi, Measure of Non-copactness for Integral Operators in Weighted Lebesgue Spaces, Nova Science Publishers, Inc, 2011.

MTH-811 Applied Dimensional Analysis and Modeling

Physical quantities and equations, Mathematical preliminaries of dimensional analysis with examples and fundamental results, Formats and classification, Dimensional systems with examples, Transformation of dimensions, Arithmetic of dimensions, Dimensional homogeneity, Structure of physical relations, Systematic Determination of Complete Set of Products of Variables with comprehensive details and examples, Dimensional Analysis; The steps of dimensional analysis and Buckingham's Pi-Theorem, Deformation of an elastic sphere striking a wall, The end game, On the utility of dimensional analysis and some difficulties and questions that arise in its application: Out-of-scale modeling, Dimensional analysis reduces the number of variables and minimizes work. An incomplete set of independent quantities may destroy the analysis, Superfluous independent quantities complicate the result unnecessarily, On the importance of simplifying assumptions, On choosing a complete set of independent variable, Dimensional Analysis in Problems Where Some Independent Quantities Have Fixed Values, Transformations, Number of Sets of Dimensionless.

Reference Books:

1. Szirtes, T., Applied Dimensional Analysis and Modeling Toronto, Ontario, Canada, 2007.
2. Sonin, A.A, The Physical Basis of Dimensional Analysis, 2nd Edition, 2001.
3. Lemons,D.S, College, B., Kansas, A student guide to: Dimensional Analysis, Cambridge University Press, 2017.

MTH-812 Dynamical Systems and Control Theory

Characterization of chaos in different systems, Bifurcations, Rossler system, mappings, Poincaré section, p-cycles, folding and stretching, Lyapunov exponents, Henon map, saddle manifolds, homoclinic tangles, and basin of attraction, Fractals and fractal dimensions with appropriate examples and its geometry Linear dynamical systems, basics, state space solutions and realizations Stability, controllability & observability, State feedback and state estimation Optimiation problems of dynamic Systems, optimization problems with path constaints, optimal feedback control Linear systems with quadratic criteria, optimal feedback control in the presence of uncertainity, Bellman's equation & dynamical programming: (a) calculus of variations, (b) computational aspects. Nonlinear system analysis: phase plane analysis, Lyapunov theory, advanced stability theory Nonlinear Control Systems Design: Feedback linearization Sliding mode control, adaptive control Adaptive control, control of multi-input physical systems,schochastic and adaptive control

Reference Books:

1. Kailath, T. Sayed, A.H., Hassibi,B., Linear Estimation, Prentice Hall, 2000.
2. Khalil,H.K., Nonlinear Systems, Prentice Hall, 2010.
3. Bellman, R., Adaptive Control Process, Princeton University Press, 2009.

MTH-813 Advanced Dynamics

Equations of dynamics and its various forms, equations of Lagrange and Euler, Jacobi's elliptic functions and the qualitative and quantitative solutions of the problem of Euler and Poisson, the problem of Lagrange and Poisson, equations of Hamilton and Appell, Hoamilton-Jacobi theorem, sepaerable systems, Holder's variational principle and its consequences.

Reference Books:

1. D.T. Geenwood, Classical Dynamics, Dover, 1997.
2. Dan B. Marghitu ,Mihai Dupac, Advanced Dynamics, Analytical and Numerical Calculations with MATLAB, Springer, 2012.

MTH-814 Statistical Methods for Data Science

In the course, the following broad areas will be covered; data analysis including descriptive statistics and data visualization probability theory including basic probability calculations, random variables, distributions statistical methods including point and interval estimates, hypothesis testing, regression probabilistic models in data science applications, for instance, Naive Bayes classifiers and topic models for text or Hidden Markov Models for sequences.

Reference Books:

1. Charles Wheelan, Naked Statistics: Stripping the Dread from the Data, W. W. Norton & Company, 2013.
2. Peter Bruce and Andrew Bruce , Practical Statistics for Data Scientists, O'Reilly Media, Inc., 2017.

MTH-815 Nonlinear Functional Analysis

This course covers fundamental techniques in nonlinear functional analysis, as well as selected applications. Topics include the contraction mapping principle, Frechet derivatives and higher derivatives of nonlinear, functions between Banach spaces, the implicit function theorem, Lyapunov-Schmidt reduction, Newton polygon method, topological degree theory, and bifurcation theory. Reiz representation theorems and Reiz potentials and its applications.

Reference Books:

1. Functional Analysis by Alexander C. R. Belton, Cambridge University Press, Cambridge, 2004 and 2006.
2. A. Ambrosetti and D. Arcoya, An introduction to nonlinear functional analysis and elliptic problems, Birkhäuser, 2011.

2. E. Kreyszig, Introductory functional analysis with applications, John Wiley & Sons, Inc., New York, 1978.

3. P. Lax, Functional analysis, John Wiley & Sons, Inc., New York, 2002.

MTH-816 Optimization Theory

Necessary and sufficient conditions for extrema, Newtonian Method, Constrained derivative (Jacobian) method and its sensitivity analysis, Lagrangian method and extension, Kuhn-Tucker conditions, Direct search method, Gradient method and reduced gradient method, separable programming, Quadratic programming. Geometric programming, linear combination method. Integer programming,, Examples, Branch and bounds algorithm, Cutting-plane alogorithm, zero-One polynomial programming, introduction to game theory.

Reference Books:

1. Rangarajan K Sundaram, A First Course in Optimization Theory. Cambridge University Press, 1996.
2. Hubertus Th Jongen ,Kxaus Meer ,Eberhard Triesch,Optimization theory. Springer, 2004.
3. G R Sinha , Ahmed Sirajuddin , Chamorshikar Rajesh , Choubey Siddharth , Choubey Abha, Modern Optimization Methods for Science, Engineering and Technology, Iop Publishing Ltd, 2020.

MTH-817	**Special Topics**

MTH-799	**MS Thesis**

MTH-899	**PhD Thesis**